ABSTRACT
        This learning module is designed to enable teachers
to help children further develop their concepts of the meaning of
numbers in our number system (including concepts of quanity,
one-to-one correspondence, number order, before-after, greater
than-equal to-less than, number patterns, and the structure of our
number system). The module contains a rationale, general objectives,
specific objectives, and a list of materials and equipment. There is
also a list of general instructions for the teacher or student
teacher. The procedures for using the module include a pre-test, a
video tape, reading, studying games, several optional activities, and
a post-test. A copy of the pre-test and its answer sheet, along with
a copy of the post-test and answer sheet, are included. The module
also contains games with tri-dominoes. A list of suggested readings
and a bibliography complete the module. (RC)

Number Patterns and Systems

Learning Activity Module IV
Developed for use in Department of Elementary Education
at the University of Florida

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Dr. J. J. Shea   Microteaching Laboratory
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University of Florida
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TITLE: NUMBER PATTERNS AND SYSTEMS

RATIONALE: One major change regarding arithmetic content is attention being given to the structure of numbers. Textbook writers now consider a knowledge of the number system to be so important that they give some work on it at each grade level throughout the elementary school. To help reason the "why" of "what is" in our base ten system teachers need to be aware of the various number systems. Likewise, they need to know ways to teach children concepts of: quantity, one-to-one correspondence, number order, before-after, greater than-less than-equal to and recognition of number patterns. Teachers also need to be aware of how children develop in their thinking and reasoning of these concepts (especially the work of Piaget on the child's conception of number).

GENERAL OBJECTIVES: To be able to help children develop further their concepts of the meaning of numbers in our number system; including concepts of quantity, one-to-one correspondence, number order, before-after, greater than-equal to-less than, number patterns, and the structure of our number system.

SPECIFIC OBJECTIVES:

1. To be able to identify and give examples of the major classifications of our number system (Natural, Whole, Integer, Rational, Irrational, Real, Imaginary, and Complex).

2. To be able to identify and illustrate one-to-one correspondence with matching, and non-matching sets.

3. To be able to illustrate at least five different patterns for any number larger than three.

4. To be able to explain Piaget's experiment and stages of the child's development in his thinking processes in respect to "Conservation of·Numberness."

5. To be able to demonstrate a way a teacher would develop the concept of quantity (e.g. "threeness") with children in the kindergarten or first grade. This would include identifying at least 3 teaching techniques that might be used, 3 teaching aides that would help concretely illustrate the concept of "quantity," and at least 3 different activities that might help children learn this concept.

MATERIALS &   1. Learning Activity Module, NUMBER PATTERNS AND SYSTEMS.
EQUIPMENT:
               2. Demonstration Classroom tape, NUMBER PATTERNS AND SYSTEMS

               3. Video Tape Recorder (audio optional)

               4. Any one or more of the resource texts listed in the reading section of the procedures and identified in the resources bibliography. (Texts are available for loan in the library.)

OPTIONAL MATERIALS:

               5. Film strip - "Thinking About Four" - by Martha Reihard and Nancy Peterson .

               6. Filmstrip Projector (Standard)

               7. Super 8mm single concept film loop projector.

               8. Super 8mm film loops:

                   One-to-one   #S-80161
                                              Encyclopedia Britannica
                   Inequalities #S-80086   Educational Corporation

               9. Audio tapes: Imperial Primary Math Skills

                   Improvement Tapes: Lessons 16 and 17 Equalities and

                   Inequalities

GENERAL INSTRUCTIONS TO PARTICIPANTS

This Module is designed to be used by one teacher or student teacher (the participant) working independently. This may take place as part of a workshop or class with two or more participants working together. You may also work on your own.

At times, the procedures will refer to a leader or director. This would be someone from whom you can get help and feedback while you are working on the module or after it has been completed. (This person may be a college methods teacher, a workshop leader, a principal or curriculum director, another teacher or another participant who is familiar with the material you are working on.) Completion of a Module may require an observer to view the participant's video tapes and discuss and/or give feedback to the participant. To complete the module follow the procedures on the next page.

Module IV   NUMBER PATTERNS AND SYSTEMS

PROCEDURES

1. **PRE-TEST**   Obtain Pre-Test IV from your leader, or use a separate sheet of
paper for your answer sheet and turn to the Pre-Test on page 7 of this
module.   Take and score the Pre-Test.   The answers are found on page 8
of this booklet.   Give yourself points as follows:

> 1.   1 @ letter = 5 points
>
> 2.   1 @ letter = 7 points
>
> 3.   1 @ criteria = 3 points
>
> 4.   1 @ pattern = 4 points
>
> 5.   Subjective Judgment 3 points total
>
> 6.   Subjective Judgment 5 points total
>
> (If your teacher or helper is available ask
> them to judge 5 and 6 for you.)

> Total - 27 points possible
>
> If you score 22 or higher you may skip Module IV.   Go on
>
> to Module V.   If not, complete the following instructions:

2. **VIDEO:**   View Video Tape IV, <u>Number Patterns and Systems</u>.

3. **READ:**   Choose one or more of the following:

Collier and Lerch (1) - pages 47 - 53

Copeland (2) - pages 57 - 67

Copeland (3) - pages 14-17, 25-26, 20, 21, and Chapter 14, pages 223-232

D'Augustine (4) - pages 31-52

Kennedy (8) - pages 6-8, 30-32, 40-42

Marks (9) - pages 57-58, 63-65, 70

Olson. (11) - pages 8-11

Shipp and Adams (13) - pages 77-83, Chapter 6, pages 149-188

4. GAME: Tri-dominoes - study the games listed on page 12 of this module and play out the games suggested with the set of tri-dominoes included in the envelope on the last page of this module. Teach one of the games to a small group of children (or play it with other module participants in simulation).
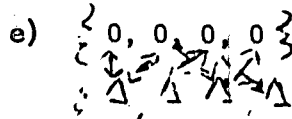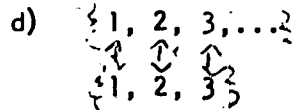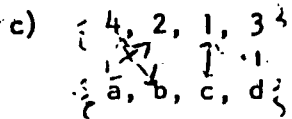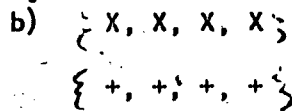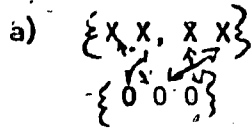
5. OPTIONAL ACTIVITIES:

   A. Filmstrip: View "Thinking about Four" by Martha Reihard and Nancy Peterson.

   B. Pattern Cards: Make your own set of pattern cards that can be used with your children.

   C. Tri-dominoes: Make your own set of tri-dominoes, following the pattern included with the game section of this module.

   D. Film loop: View the 8mm film loops 1) One-to-One (Encyclopedia Britannica Educational Corporation #S-80161). Study the guide that comes with the film loop. Concepts: One-to-one correspondence is a more basic idea in mathematics than counting, and patterns in numbers. 2) Inequalities #S-80086

   E. Audio Tapes: Select the Imperial Primary Math Skills Improvement tape lessons 16 and 17 on Equalities and Inequalities. These tapes are for your children but you should be familiar with their content and method of presentation. Review the Teacher's Manual and the child's worksheet that corresponds to the tape, listen to the tape and complete the exercises suggested.

6. <u>POST-TEST</u> Turn to the post-test section of the module, page 9 . Take

the post-test IV like you took the pre-test. Check your own post-test

with the answers given on page 11 . Score yourself the same way you did

for the pre-test. If you score 22 or higher, go on to Module V. If not,

review the video tape for the concepts, information, or skills you are

not yet sure of on the post-test. Also, choose another selection from

the list of optional readings. Then try the post-test again (or an

alternate post-test, if available from your leader). If you do not pass

it this time, set up a conference with your leader.

Pre-test IV

Number Patterns and Systems

1. Which of the following diagrams represents one-to-one correspondence?

   a) {X X, X X}
      {0 0 0}

   b) { X, X, X, X }
      { +, +, +, + }

   c) { 4, 2, 1, 3 }
      { a, b, c, d }

   d) {1, 2, 3,...}
      {1, 2, 3}

   e) {0, 0, 0, 0}
      { △, △, △, △ }

2. Match the Number System with the example.

   _____ a. Whole Numbers
   _____ b. Integers
   _____ c. Real Number System
   _____ d. Imaginary Numbers
   _____ e. Rational Numbers
   _____ f. Irrational Numbers
   _____ g. Natural Numbers

   t. all fractions that have a terminating or repeating decimal equivalent.
   u.   1, 2, 3,...
   v. 0, 1, 2, 3,...
   w. includes rational and irrational no's.
   x. all numbers that produce a non-terminating non-repeating decimal.
   y. ..., -3, -2, -1, 0, +1, +2,...
   z. √-1

3. Make up a drawing of a sample problem that would help primary level children use: one-to-one correspondence, identify which of the two groups is greater than the other, and could answer your question. "How much greater?"

4. Draw 4 <u>different</u> patterns of 5 dots. (Make sure one pattern is not just another pattern turned around.)

5. Explain Piaget's experiment on concept development in "Conservation of Number."

6. Teaching Problem: Can you explain how you as a teacher would develop the concept of quantity (for example "fiveness") with children in kindergarten or first grade. What teaching techniques would be used with the teaching aides you would employ? Do not make a list, rather explain in essay form how you would teach a concept, step by step.

ANSWER SHEET FOR PRE-TEST   IV

NUMBER PATTERNS AND SYSTEMS

1. C

2. A. - V   D. - Z   G. - U
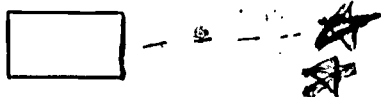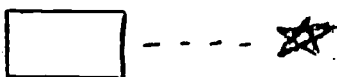   B. - Y   E. - T
   C. - W   F. - X

3. Any drawing that includes:

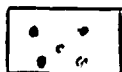   A. At least two groups of objects

   B. Lines drawn from each object in one group to an
      object in the second group.

   C. If they are not equal groups it is obvious that one or
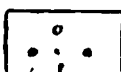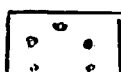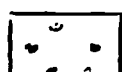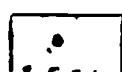   more objects have no partner in the other group.

      E.G.

4. Here are 10 possible answers:

   Dominoe   Cross   House   Pentagon   4 - 1
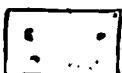
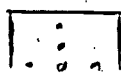   "U"   "T"   3 - 2   "Chair" or   "V"
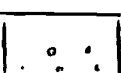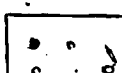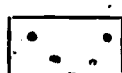   "Dipper"
                                   or "Hat"

   ETC.   There are many more.
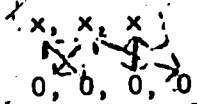
5. Conservation of Number - The idea of relation is important. Some of the
   first experiences in mathematics for children involve the comparison or
   relation of two quantities. Are they "the same as" or "equal" or is one
   "greater" than the other or "smaller" than the other? In this activity
   the child is asked these questions for different arrangements of two equal
   groups of blocks to determine if he understands the quantitative relation
   separate from the group's spacial arrangement.

6. Review tape for answer.

Post-test IV

Number Patterns and Systems
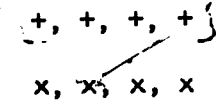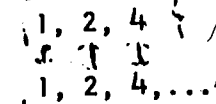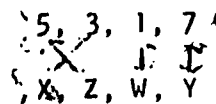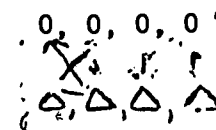
1. Which of the following diagrams represents one-to-one correspondence?

a.) x, x, x
 0, 0, 0, 0

b.) +, +, +, +
 x, x, x, x

c.) 1, 2, 4
 1, 2, 4,...

d.) 5, 3, 1, 7
 X, Z, W, Y

e.) 0, 0, 0, 0
 △, △, △, △

2. Draw 4 **different** patterns of 6 dots. (Make sure one pattern is not just another pattern turned around).

3. Match the Number System with the example.

_____ a. Imaginary Numbers

_____ b. Integers

_____ c. Irrational Numbers

_____ d. Natural Numbers

_____ e. Rational Numbers

_____ f. Real Numbers

_____ g. Whole Numbers

t. all fractions that have a terminating or a repeating decimal equivalent.

u. 1, 2, 3,...

v. 0, 1, 2, 3,...

w. include rational and irrational numbers

x. all numbers that produce a non-terminating non-repeating decimal.

y. .., -3, -2, -1, 0, +1, +2,...

z. $\sqrt{-1}$

4. Make up a drawing of a sample problem that would help primary level children use: one-to-one correspondence, identify which of the two groups is less than the other, and could answer your question "How much less?"

5. Explain Piaget's experiment on concept development in "Conservation of Number."

6. Teaching Problem: Explain how you as a teacher would develop the
    concept of quantity (for example "fourness") with children in
    kindergarten or first grade. What teaching techniques would be used
    with the teaching aids you would employ? Don't make a list, but
    rather explain in essay form how you would teach the concept, step
    by step.

Answer Sheet for Post-Test IV

NUMBER PATTERNS AND SYSTEMS

1. D

2. Here are 10 possible answers. There are many more.

"Dominoe"   "T"   "X"   2 - 4

"Y"   "V"   "U"   "Chair" or
                          "dipper" or "hat"

"Hexagon"   "Triangle"
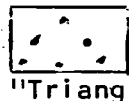
3. a. ___z___   d. ___u___   g. ___v___

   b. ___y___   e. ___t___

   c. ___x___   f. ___w___

4. Any drawing that includes:

   A. At least two groups of objects
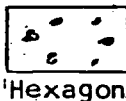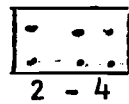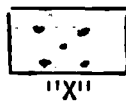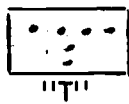
   B. Lines drawn from each object in one group to an object in the second group.

   C. If they are not equal groups it is obvious that one or more objects have
      no partner in the other group.

   E.G.

5. Conservation of Number - The idea of _Relation_ is important. Some of the first
   experiences in mathematics for children involve the comparison or relation of two
   quantities. Are they "the same as" or "Equal" or is one "greater" than the
   other or "smaller" than the other? In this activity the child is asked these
   questions for different arrangements of two equal groups of blocks to determine
   if he understands the quantitative relation separate from the group's spacial
   arrangement.

6. Review tape for answer.

13

Games to Play with Tri-Dominoes

Game 1. __Match the Dots__   (One-to-One Correspondence)

Object: Match the dots on the last tri-dominoe played.
First person to use up all his tri-dominoes wins. (Goes out first.)

Start: Turn all tri-dominoes face down. Each player draws
five, laying them face up in front of him. Person with lowest total
of dots on any one of the tri-dominoes is first.

Play: Player to the left plays next, matching one side of
the last played tri-dominoe with one of his. The next person on the
left plays on the last played tri-dominoe, etc.

Bone Pile: If a player cannot match on his turn, he must take
new tri-dominoes from the bone pile (the remaining undrawn face down pieces)
until he does make a match or until the pile is empty. If he still cannot play
he loses his turn to the next player.

Matching Two Sides: Sometimes it may be possible or necessary
to match two tri-dominoes with one. For example:
If a player had a tri-dominoe which matched the one dot
of the last played piece he would also
need to match the two dots of the starting piece
in order to play his piece in the space shown.
Otherwise, he can only play on the four-dot side.
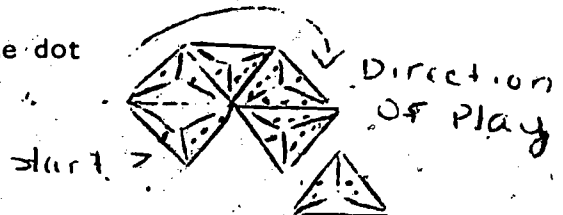
Holes (Matching All Three Sides): Two other ways to end the game
without going out may occur if holes are created in the pattern of the tri-dominoes
as illustrated below. Player with the fewest number of tri-dominoes left in his
hand is the winner.

14
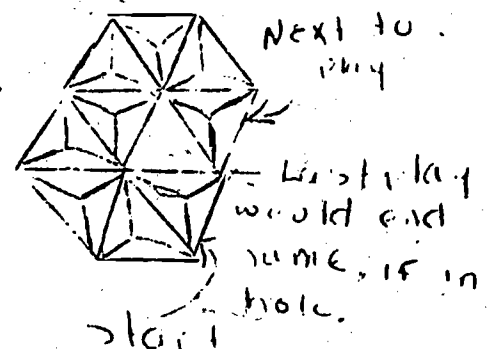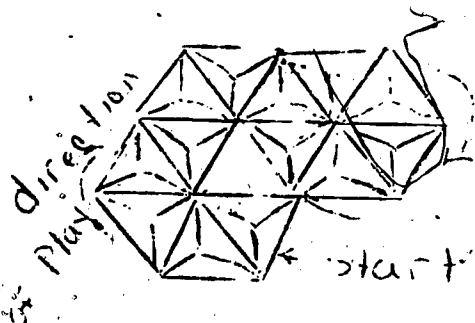
Game 2. Make Four   (Simple addition Combinations thru 4+4)

Object: Add combinations of dots to make a sum of four. First one who goes out wins.

Start: Same as in game number one above.

Play: Instead of matching dots, face together dots whose sums equals four: blank with four dots, three with one, two plus two. If a combination of four cannot be made the player must draw from the bone pile as in game one. A play can be made as long as a player can get a sum of four on one side of the tri-dominoe even if he places his piece next to two tri-dominoes or in a hole (three tri-dominoes).

Strategy: If you have a choice of pieces to play, look ahead to your left to see which one would be less advantageous for your opponent to play on. Can you make him dig in the bone pile?

Game 3. Addition   (Addition and Subtraction thru two figures)

Objective: Have highest total of points from adding combinations of dots.

Start: Same as in game two.

Play: With each new game players take turns in starting. First player draws an extra tri-dominoe from the bone pile, places it face up in the center of the playing area, and begins his first play on it. Any tri-dominoe can be played on the last one played. When play is made the points on the matching sides are added, eg. 3+1, 4+3, etc. With the highest addition 4+4. (Lowest 0+0) Scores are kept for each player and added with each turn. When someone goes out the game is over, person with the highest score is the winner.

Options: Player may choose to draw from bone pile as many times as he wishes for a higher possible piece instead of playing one from his hand. However, when another player goes out, each remaining player loses 5 points for each piece he has not played.

Bonus Option: The player who goes out get a pre-determined number of points for going out. (Usually not more than 10.)

CAN YOU THINK OF OTHER GAMES YOU CAN PLAY WITH TRI-DOMINOES THAT GIVE PRACTICE IN MATHEMATICAL SKILLS?

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cut
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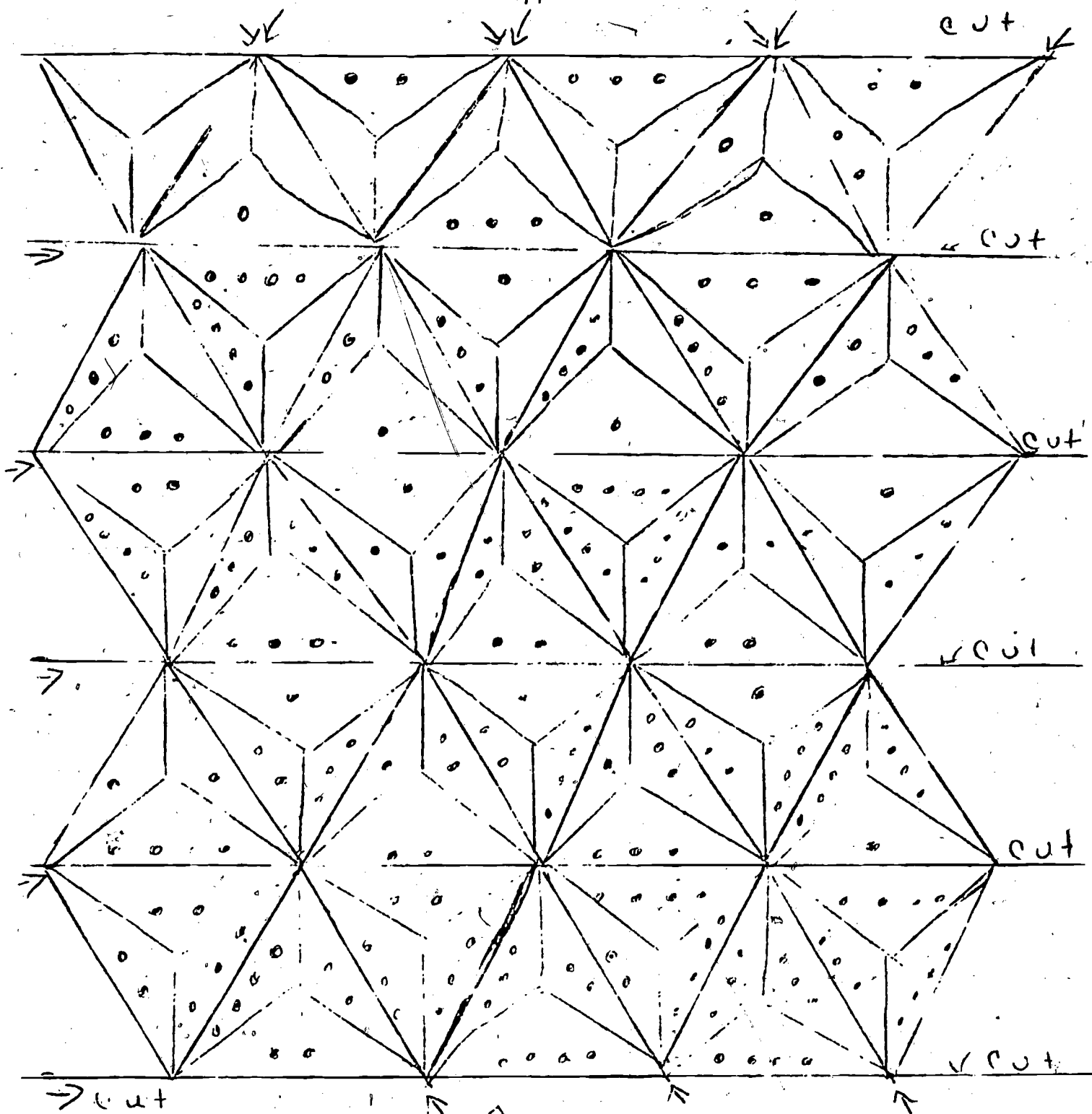
TRI-DOMINOS   Dr. Bernard

SUGGESTED READINGS OF CONTENT TEXTS ON UNDERSTANDING NEW MATH CONCEPTS. (NOT

ON "HOW TO TEACH")

Adler, Irving. The New Math, New York: The John Day Company, 1958.

Armstrong, James W. Mathematics for Elementary School Teachers, New York:
      Harper and Row Publishing Company, 1968.

Bauwsma, Ward D., Clyde G. Corle and Davis F. Clemson, Jr. Basic Mathematics
      For Elementary Teachers, New York: The Ronald Press Company, 1967.

Boyer, Lee Emerson. An Introduction to Math for Teachers, New York: Henery
      Holt and Company, 1967.

Brueckner, Leo J. and Foster E. Grossnickle. Discovering Meanings in Ele-
      mentary School Mathematics, New York: Holt, Rinehart and Winston, 1963.

Bureau of Elementary Curriculum Development. Number and Numeration, Albany:
      The University of New York, 1963.

Fogal, Carroll E. Exploring Numbers and Forms, Alachua County ETV Study Guide.

Gechtman, Murray and Hardesty, James. Arithmetic: Concepts and Skills,
      New York: The MacMillan Company, 1968.

Kelly, John and Richert, Donald. Elementary Mathematics For Teachers,
      San Francisco: Holden-Day, 1970.

Meserve, Bruce E. and Max A. Sobel. Introduction to Mathematics, Englewood
      Cliffs, New Jersey: Prentice-Hall, Incompany, 1964.

Mueller, Francis J. Arithmetic, Its Structure and Concepts,
      Englewood Cliffs, New Jersey: Prentice-Hall, Incompany, 1964.

Nichols, Eugene D. and others. Elementary Mathematics, New York:
      Holt, Rienhart and Winston, Incompany, 1966.

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3. _____. Mathematics and the Elementary Teacher, Philadelphia: W. B. Saunders Company, 1968.

4. D'Augustine, Charles H. Multiple Methods of Teaching Mathematics in the Elementary School, New York: Harper and Row, 1968.

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7. Heddens, James W. Today's Mathematics: A Guide to Concepts and Methods in Elementary School Mathematics, Revised, Chicago: Science Research Association, 1970.

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9. Marks, John L.; Purdy, Richard C. and Kinney, Lucien B. Teaching Elementary School Mathematics for Understanding, Third Edition, New York: McGraw Hill Book Company, 1970.

10. May, Lola June, Teaching Mathematics in the Elementary School, New York: The Free Press (Collier-MacMillan), 1970.

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